Name ___________________________ID No. _________________________

- Results already derived in the textbook may be used as long as the results are clearly referenced (page number, formula number, etc.).
- Fundamental constants are listed near the back of the textbook.
- All numeric answers should reach an accuracy better than 10%.
- All frequencies referred to in this exam are angular frequencies.

1. (25 points) The Ca^+ ion clock.

a) (10 points) Energy levels of $^{40}Ca^+$ (nuclear spin I = 0) are shown in Fig. 1.

i. Identify all E1 and E2 type transitions among the levels by linking the initial and final states of a transition with a line, and label the line with the type of transitions: E1 or E2.

ii. Identify the clock transition(s). The clock transition(s) should have a narrow linewidth and should be insensitive to the magnetic-field noise. The clock frequency can be that of a single transition, or a linear combination of frequencies of multiple transitions.

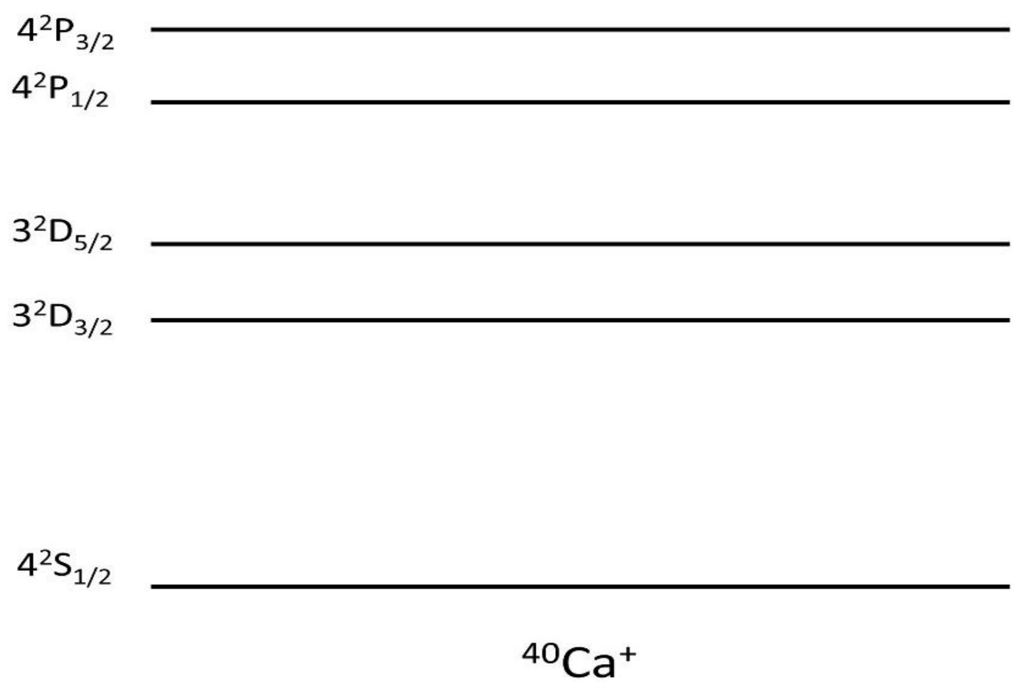

Figure 1

b) (15 points) Now consider the $^{43}Ca^{+}$ ion (nuclear spin I = 7/2)

i. Draw the hyperfine structures of $4^2S_{1/2}$ and $4^2P_{3/2}$ levels.

ii. The hyperfine coefficients for the $4^2S_{1/2}$ and $4^2P_{3/2}$ levels are A_1 = -806 MHz, A_2 = -30 MHz, B = 151 MHz. Calculate the hyperfine splittings of the $4^2S_{1/2}$ and $4^2P_{3/2}$ levels.

iii. Identify the clock transition(s).

2. (10 points) See Fig. 6.10 of the textbook (Pg. 110), on **the Zeeman effect on the hyperfine structure of the ground level of hydrogen 1s $^2S_{1/2}$**. It turns out that the top-two energy levels will cross each other at a very high magnetic field. Calculate this magnetic field of the crossing point. (hint: the g factor of the proton is $g_{proton} = 5.8$)

3. **(15 points)** Consider an **atomic fountain clock** shown in Fig. 9.16 of the textbook (Pg. 203), cold cesium atoms are launched up into free fall trajectories. They first pass through the microwave cavity on the way up and then the second time as they fall back through the cavity. Then the transition probability is measured. The ground-level hyperfine splitting provides the clock transition at 9.2 GHz. The Ramsey-fringe clock signal is shown in Fig. 2(a) (The figure is on next page).

   a) (10 points) How would the measurement be modified in order to obtain the clock signal shown in Fig. 2(b)? In the modified measurement, how would Equation (7.52) of the textbook (Pg. 133) be modified?

   b) (5 points) Now suppose the atoms are launched up with an initial velocity that is twice as fast as in the case shown in Fig. 2(a). Draw the clock signal of this case in Fig. 2(c).

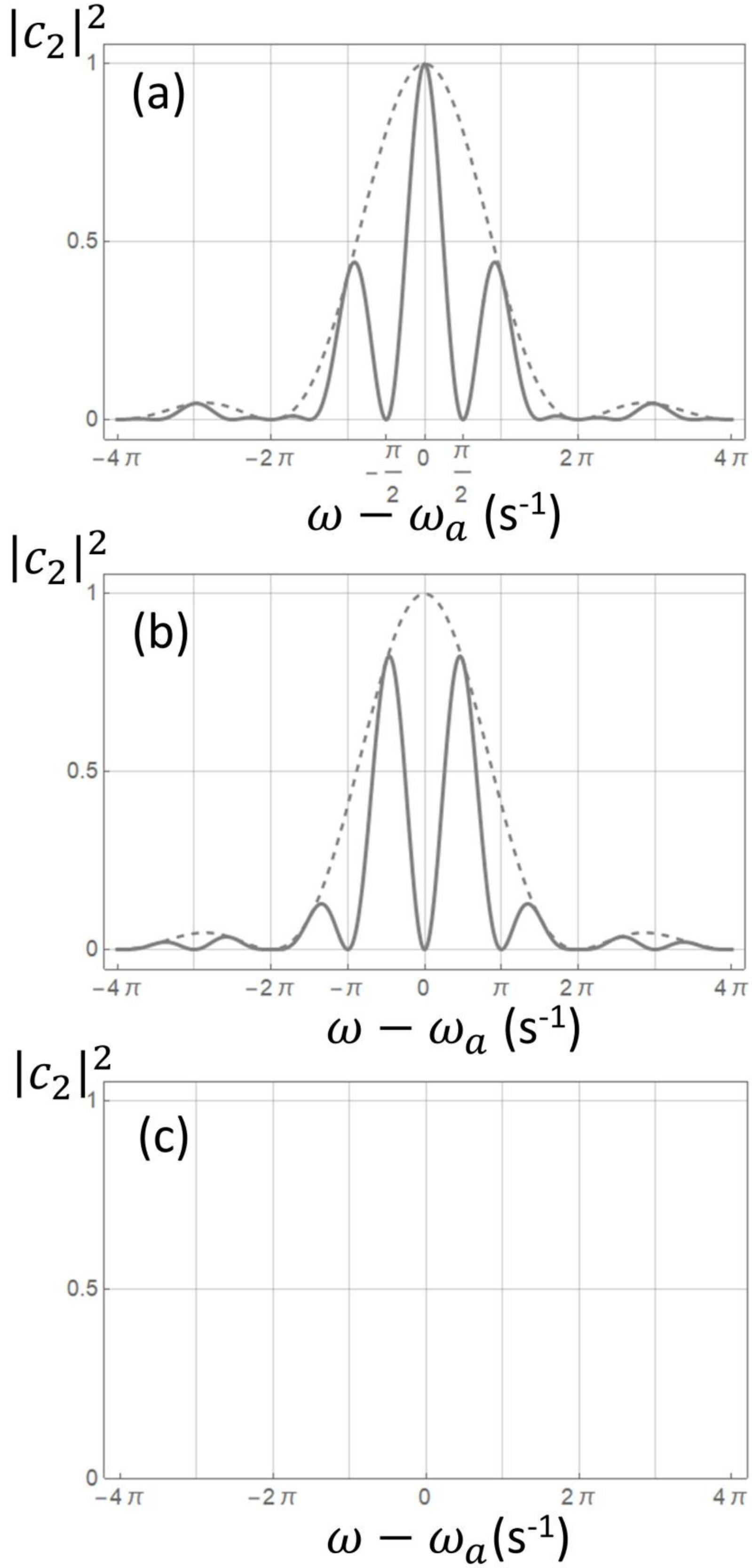
|c2|2
(a)
1
0.5
0
−4 π
−2 π
−π/2
0
π/2
2 π
4 π
ω − ωa (s-1)
|c2|2
(b)
1
0.5
0
−4 π
−2 π
−π
0
π
2 π
4 π
ω − ωa (s-1)
|c2|2
(c)
1
0.5
0
−4 π
−2 π
0
2 π
4 π
ω − ωa(s-1)

Figure 2