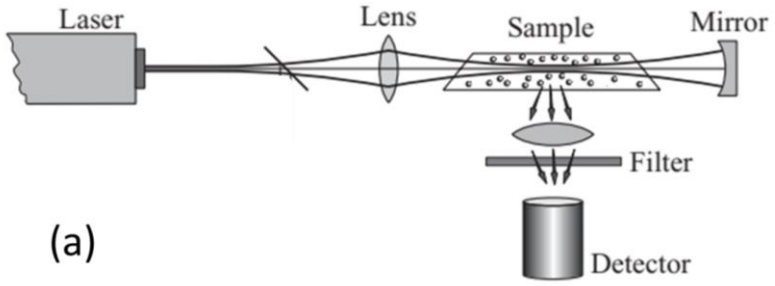
1. (25 points) Consider the Doppler-free two-photon laser spectroscopy of the 1S-2S transition of tritium (T or ^{3}H, I = 1/2) atoms (Fig. (a)). A thermal gas of T atoms at room temperature (300K) is probed by the laser. The diameter of the laser is 0.2 mm. The length of the region where the atoms and laser beam are overlapped is 0.25 m. (The linewidth of the laser is negligible in this problem.)



a) (5 points) Draw the 1S and 2S hyperfine levels of the T atom in an external B-field. Identify all two-photon transitions whose resonant frequencies are insensitive to magnetic field.

b) (5 points) Calculate the ratio of the volume shifts of the 1S level and 2S level.

c) (5 points) The life time of the 2S level of the tritium atom is very long. However, by applying an E-field during detection, the life time of the 2S state can be shortened, which will speed up the emission of fluorescence from the 2S state. Explain why.

d) (5 points) For the tritium Doppler-free 1S-2S two-photon spectroscopy, what is the wavelength of the laser? (Keep 6 significant digits in your answer. In the H Doppler-free 1S-2S two-photon spectroscopy, the laser wavelength is 243.134nm.)

e) (5 points) Identify the main mechanism that determines the width of the spectroscopic signal (Only the effects due to the motion of the atoms need to be considered. Effects due to collisions and stray E-fields, etc. are negligible). Estimate the width in the unit of MHz.

3. (15 points) We aim to levitate a ^{87}Rb atom against gravity in a laboratory. The atom is in the $5s\ ^2S_{1/2}$, F = 2 level.

a) (2 points) If a magnetic field trap is used to levitate the atom, in which M_F state should the atom be placed so that it needs the least amount of B-field gradient? Here the B field direction is defined as the quantization axis.

F = 2, M_F = _____

b) (5 points) In the above M_F state, calculate the field gradient dB/dz needed in order to levitate the atom.

c) (3 points) If optical dipole force is used to trap the atom at the focal point of a laser beam, which of the following wavelengths can be used for the laser, 777 nm or 783 nm? These wavelengths are chosen to be ± 3 nm away from the $5s\ ^2S_{1/2}$, F = 2 → $5p\ ^2P_{3/2}$, F = 3 transition. This transition has a resonance wavelength of 780 nm, a natural linewidth of $\Gamma = 2\pi \times 6\text{MHz}$ and a saturation intensity of $I_{sat} = 1.7\ \text{mW/cm}^2$.

d) (5 points) Calculate the gradient of the laser intensity dI/dz needed in order to levitate the atom.